Computer Optics Vol. 2, No. 1, pp. 11–16, 1990
Printed in Great Britain

0955–355X/90 \$10.00 + 0.00

INFORMATION PROPERTIES OF A ZONE PLATE

I. V. Minin and O. V. Minin

Abstract—The paper reports a study of the information properties of a zone lens antenna. The focusing properties for a substantially off-axis point source are investigated. The number of image elements in a plane frame and on the best focusing surface are determined and the profile of this surface is evaluated. Estimates are given for the operating range and Q-factor of a zone plate.

The simplest focusing element of computer optics is a Fresnel (or Rayleigh–Wood) zone plate. The principle underlying its operation is that it encodes each wavefront phase in the form of certain phase retardations. Applications of such elements span from X-rays to centimetre waves [1]. There are also suggestions for using them in special telescopes [2].

The focusing properties and frequency characteristics of zone plates operating in axially symmetric beams have been extensively studied for both plane [3] and spherical [4] incident wavefronts. However, the information capacity of zone plate imagery remains an open question. It is controlled primarily by the number of elements resolved within a frame, the image quality of an off-axis source, the frequency range in which the zone plate can operate, and such.

The main purpose of this work is to study the capabilities of a high-power zone plate for transmission of information, to determine its field of view, to evaluate the number of image elements within a frame, and to relate these characteristics to the frequency properties of the plate. The studies of the early 1980s are also reported.

Design relationships

The geometry of the problem is obvious from Fig. 1. The zone plate is centred at the origin, the $0Z$ is being normal to the plate and directed along the optical axis of the zone plate antenna. The zone plate is designed for an axial source at $Z_A = -A < 0$, the focal point lying at $Z_B = B > 0$. The radii of zones are given by the formula [4]

$$R_n = \left[\frac{1}{4}\left(A + B + \frac{n\lambda_0}{2} + \frac{A^2 - B^2}{A + B + n\lambda_0/2}\right)^2 - A^2\right]^{1/2}, \tag{1}$$

where λ_0 is the reference wavelength in the design of the zone plate, which in general differs from the actual wavelength diffracted at the plate. Given a plate of diameter D, the greatest number $n_{\max} = N$ is selected such that $R_N \geqslant D/2$.

In the plane $Z = Z_A$, L coherent sources are given: they occur along a line, parallel to the $0X$ axis, at points $P_l'(-h_l', 0, z_A)$, where $l = 1, 2, \ldots, L$; and h_l' is the height measured along the negative direction of $0X$ (Fig. 1 shows only one source). The position of the image of P_l' is specified in two ways: either as a point $P_l(h_l, 0, z_B)$ conjugate to P_l' with $h_l = (B/A)h_l'$; or on an arc FF''—this being the curve of best focusing. In the latter case, a local system of rectangular Cartesian coordinates (x'', y'', z'') is chosen with the origin at the point where arc FF'' and line $P_l'P_l$ intersect so that $0Y''$ is along $0Y$, $0Z''$ is along $P_l'P_l$, and $0X''$ axis is tangent to arc FF''. Then the source image P_l' becomes $P_l''(x_l'', 0, z_l'')$.

The field of diffraction is determined numerically, with the aid of the Fresnel–Kirchhoff integral represented in the form

$$U(P) = c\sum_{l,n}\int_{R_{n-1}}^{R_n} \rho\, \mathrm{d}\rho \int_0^{2\pi} f \cdot \exp(i2\pi\psi)\, \mathrm{d}\varphi, \tag{2}$$

where

$$f = \left(\frac{z_l'}{r_l'} - \frac{z_l}{r_l}\right)\frac{1}{r_l' r_l},$$

$$\psi = (r_l' + r_l)/\lambda,$$

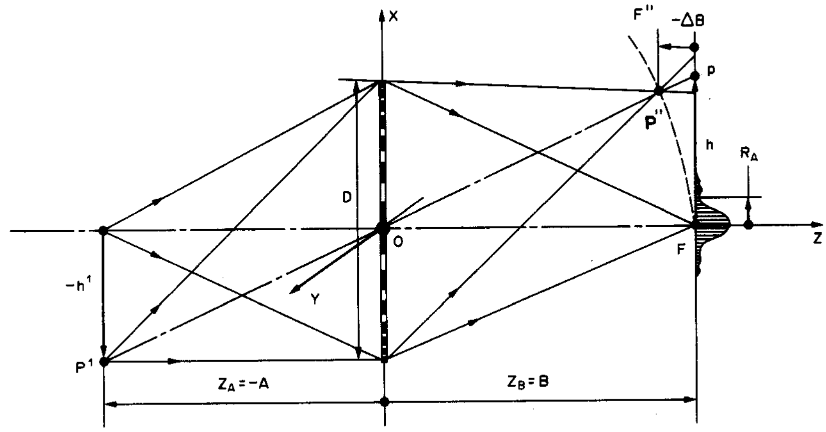

Fig. 1. Notation for a zone plate.

ρ and φ are the radius and angle in the polar system of coordinates embedded in the plane $Z = 0$, and r_l' and r_l represent the distance from the point $(\rho \cos \varphi, \rho \sin \varphi, 0)$ to P_l' and P_l, respectively.

c is the normalization constant

$n = N, N-2, N-4, \ldots, q$, where q is 1 or 0 for N even or odd, respectively.

The field at point P_l'' is computed in a similar manner by substituting P_l'' for P_l in (2).

The calculation technique is due to Baibulatov *et al.* [4]. Comparison of the calculated data with the experimental is in terms of intensity of the diffracted field $W = U \cdot U^*$.

Experimental

A zone plate designed by Eq. (1) for $D = 200\lambda_0$, and $B = A = 2D$ was made of foil-clad paper-based laminate as outlined in Baibulatov *et al.* [4].

The generator of electromagnetic oscillations was a backward-wave tube loaded onto a 1.6×0.84 mm^2 waveguide. The exit section of the waveguide served as the point source of EM radiation.

The intensity distribution of the diffracted EM field in free space was scanned by means of an open 1.6×0.8 mm waveguide loaded onto a Schottky barrier detector whose signal was amplified and recorded. The receiver was moved in various directions by a micrometre drive mounted on an optical rack. Experimental distributions of intensity as a function of coordinates were obtained by redirecting the open section of the receiving waveguide with respect to the zone plate.

SINGLE POINT SOURCE

In order to estimate the field of view and the number of image elements per frame for a zone plate antenna we carried out a number of computational and physical experiments concerning the image quality for a point source. From the problem geometry it is clear that as the point source moves away from the optical axis, its image does not move in a plane but rather goes along a certain surface (FF'' in Fig. 1) later referred to as the best focusing surface (BFS).

To a first approximation, the profile of the BFS generatrix may be evaluated from the condition that the source-to-image optical paths through the centre of the zone plate and through the edge of the nth Fresnel zone are equal. Observing the geometry of the problem we may write

$$\sqrt{(x_1 - x_n)^2 + z_1^2} + \sqrt{(x_2 - x_n)^2 + z_2^2} = \sqrt{(z_2 - z_1)^2 + (x_2 - x_1)^2} + n\lambda/2, \qquad x_1 z_2 = x_2 z_1 \quad (3)$$

where (x_1, x_2) and (x_2, z_2) are the coordinates of the source and its image respectively, and x_n is

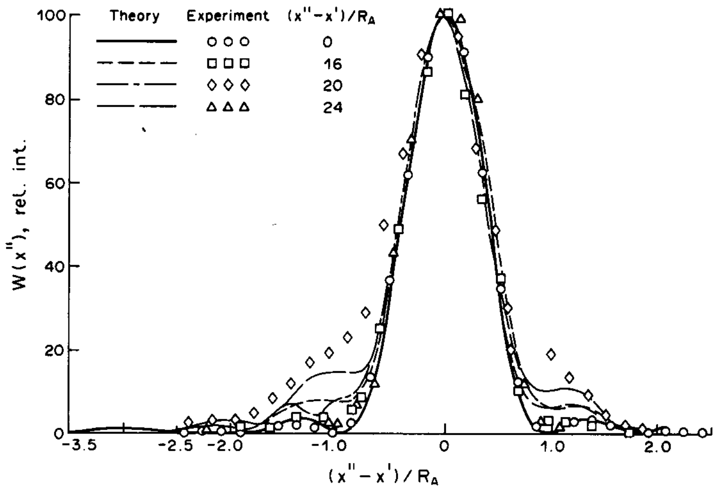

Fig. 2. Intensity distribution for an off-axial source on the best focusing surface.

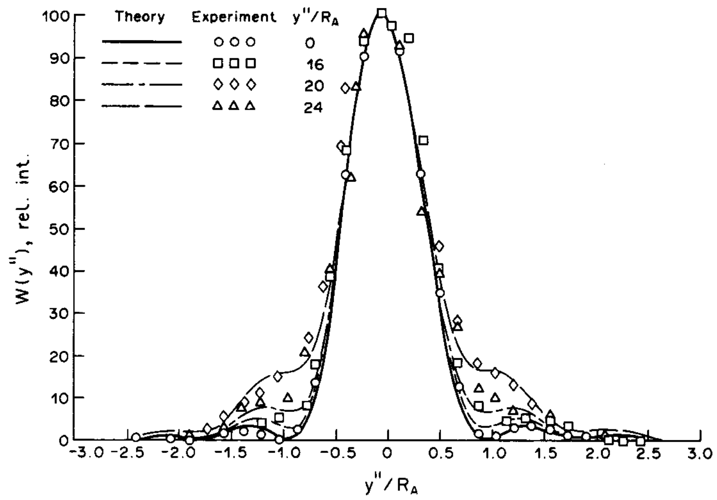

Fig. 3. Intensity distribution for an off-axial source on the best focusing surface.

the distance from the centre of the zone plate to a point on the boundary of the nth Fresnel zone. Without any loss of problem generality we assumed that $y_1 = y_2 = 0$. The system of equations (3) was solved numerically, the values of (x_2, z_2) for the extreme values of n, i.e. $n = 1$ and $n = N$, having been averaged in order to achieve a better fit with the exact solution of Eq. (2). The values evaluated in this way were taken as the initial approximation in deriving the accurate values by Eq. (2).

Results

For $D \geqslant 200\lambda$, $A = B = 2D$, we evaluated sections through the diffraction pattern of a single source ($L = 1$) as $W(x'')$ at $y = 0$ and $W(y'')$ at $x'' = x''_0$, i.e. in the surface of best focusing. The measurements were made at various ratios of $h = 0$, 16, 20, 24 to the Airy radius R_A. The results are plotted in Figs 2 and 3, every point being an average of 15–20 measurements. The theoretical profiles were derived with Eq. (2). The arguments of the functions $W(x'')$ and $W(y'')$ were taken relative to the image centre located at point $P'(x''_0, 0, z''_0)$, where x''_0, z''_0 are the coordinates of a point where line $P'P$ intersects arc FF'' at a given $h' = (B/A)h$.

As can be seen there is a satisfactory agreement between theory and experiment—the root mean square deviation of the whole run of experiments from the computational data is about 3%.

These measurements have shown that a plate antenna with aperture ratio $D/B \sim 1/2$ and aperture

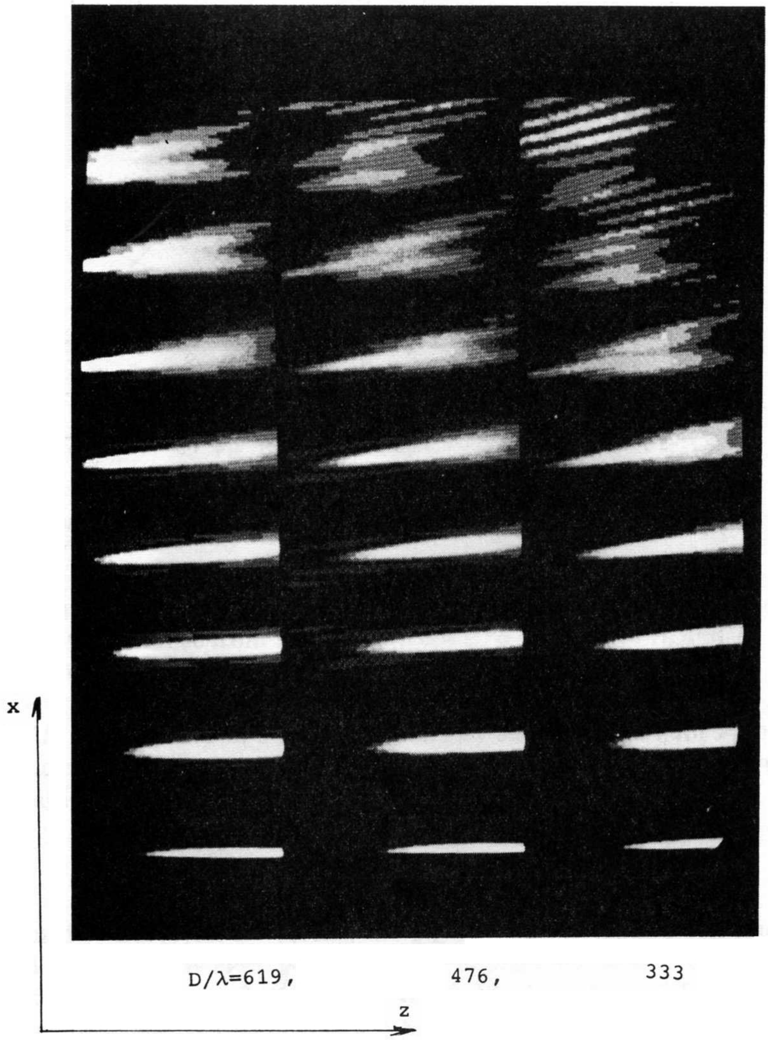

Fig. 4. Visible domains of converging beams for the apertures $D/\lambda = 619$, 476 and 333.

$D/\lambda \approx 200$ is capable of resolving about 50×50 diffraction elements in the frame lying on the best focusing surface. A special study has indicated that for a plane image frame the number of resolved elements reduces to about 30×30 [5].

Numerical and physical experiments were also carried out for wavelengths adjacent to the design value of λ_0 by $\pm 10\%$. The position of the focal plane (and the best focusing surface) was found to shift as a function of λ [4]; other findings also confirm those of Baibulatov *et al.* [4]. In the frequency range under study, the best focusing surface did not suffer any variation in shape.

SYSTEM OF COHERENT POINT SOURCES

In order to investigate the effect of interference in the system of coherent point sources on the BFS profile and on the frame resolution we numerically solved Eq. (2) for $L = 9$ sources whose height were taken to alter with a uniform step as

$$h_l = (B/A) \cdot 8R_A(l-1), \tag{4}$$

where $R_A = 1.2197\lambda$, B/D is the Airy radius, and $l = 1, \ldots, 9$.

The field intensity $W = |U|^2$ was computed at various points of the XOY plane and visualized on a half-tone visual display. The resulting visualized domain of converging waves is shown in Fig. 4 (the beam for the source $l = 1$ is shown for the half-space $x \geqslant 0$ only).

This analysis also confirmed the proposition that the field of view of a zone plate antenna can be expanded by increasing D and retaining the aperture ratio $D/B = 1/2$.

From Fig. 4 it follows that at $D \approx 300\lambda$ the image of the source $l = 5$ still remains satisfactory at a certain arc of best focusing. Hence, the greatest number of image elements resolved by the Rayleigh criterion in a curvilinear string is about 64.

If D is increased by a factor of two the number of resolved elements goes up to 80–90, however this antenna is rather large.

The computations done for $D = 200\lambda$ give about 50 image elements in a string.

Figure 4 indicates also that the best focusing surface is appreciably curved towards the zone plate. The profile of arc FF'' (see Fig. 1) may be approximately described by the relation

$$\Delta B = Hh^{3/2}, \tag{4}$$

where ΔB is the sag of arc FF'' from the plane $Z = Z_B$ (mm), and h is the height, also in mm. Estimations of H by virtue of this relation, for $D/\lambda = 300$, 450 and 600, obtained from Fig. 4 for $l = 5$, 6 and 7 yield values close to 1.9×10^{-2}, 1.7×10^{-2} and $1.6 \times 10^{-2}\ \mathrm{mm}^{-1/2}$, respectively.

Operating range

Baibulatov *et al.* [4] have reported a detailed study of the frequency properties of zone plate antennae for an axial source. A satisfactory analytical description has been derived for the frequency characteristic on the basis of the equation for Fresnel zone radii

$$B(\lambda) = \frac{A^2 + (A + n\lambda/2) - 2\sqrt{A^2 + R_n^2}(A + n\lambda/2)}{2\sqrt{A^2 + R_n^2} - 2A - n\lambda}. \tag{5}$$

With this expression at hand one may readily estimate the operating range of a zone plate antenna designed to transform a divergent wavefront into a convergent one. The shortest feasible wavelength, $\lambda_{\min}$, may be determined from the condition $B \to \infty$. Letting the denominator of (5) be zero yields

$$\lambda_{\min} = \frac{2}{n}(\sqrt{A^2 + R_n^2} - A). \tag{6}$$

Likewise, at the maximum feasible wavelength $B \to 0$, then by virtue of (5) and (6)

$$\lambda_{\max} = \lambda_{\min} + \sqrt{\lambda_{\min}^2 + 2A(A + \sqrt{A^2 + R_n^2})\frac{4}{n^2}}.$$

Observing that usually $\lambda_{\min} \ll \lambda_{\max}$, we have

$$\lambda_{\max} \approx \frac{2}{n}[2A(A + \sqrt{A^2 + R_n^2})]^{1/2}. \tag{7}$$

Equations (6) and (7) define the operating range of a zone plate. For a zone plate antenna that transforms an incident plane wavefront into a converging spherical wavefront, one can readily obtain with a similar argument that $\lambda_{\min} = 0$ and $\lambda_{\max} = 2R_n/n$. This implies a wide passband with this type of zone plate.

Quality factor

Consider the paths of rays from an axial point source through the outer zone of a zone plate antenna. The path length

$$m = m_1 + m_2 = \sqrt{A^2 + D^2}/4 + \sqrt{B^2 + D^2}/4$$

accommodates n_0 wavelengths, i.e. $n_0 = (m_1 + m_2)/\lambda_0$. When λ_0 is slightly detuned by $\Delta\lambda$, n_0 changes by Δn. Let $\Delta n = 1$, then

$$\lambda = \lambda_0 + \Delta\lambda,$$

$$n = n_0 - \Delta n,$$

$$n_0 - 1 = (m_1 + m_2)/(\lambda_0 + \Delta\lambda),$$
$$n_0 = (m_1 + m_2)/\lambda_0,$$

whence $\Delta\lambda/\lambda_0 = \lambda_0/(m_1 + m_2 - \lambda_0)$. Observing that $\lambda = c/f$ and $d\lambda = -cdf/f^2$ we have

$$\frac{\Delta f}{f_0} = -\frac{c}{f_0(m_1 + m_2) - c}.$$

Thus, for a path difference of λ_0, the frequency deviation is $\Delta f \approx -c/(m_1 + m_2)$. Let $f_0 = 150$ GHz, $m_1 = m_2 = 50$ cm, then $\Delta f \approx 300$ MHz. Accordingly, the efficient Q-factor is

$$Q = \frac{f_0}{\Delta f} = \frac{f_0(m_1 + m_2)}{c} - 1 \approx \frac{m_1 + m_2}{\lambda_0} = 500.$$

An obvious implication is that the quality factor of a zone plane antenna is proportional to its aperture ratio, i.e. $Q \sim D/\lambda$, since $m_{1,2} \sim D$. Long-focus systems seem to be more attractive from this standpoint, as $Q \sim A$ and B.

A similar consideration for an off-axis source, say shifting to the margin of field of view where $\Delta x \sim D/4$, and $A \sim B \sim 2D$ yields about the same Q-factor as in the case with an axial source. An important conclusion on the Q-factor of a zone lens is that it is almost independent of the position in the focal plane of the EM point source.

CONCLUSION

Investigations of an axially symmetric zone plate revealed a number of properties as follows:

(i) The zone plate retains its focusing properties for an off-axial source and may be used as a focusing element in a wide range of frequencies.

(ii) A zone plate with aperture ratio $\sim 1/2$ and aperture ~ 200 resolves about 50×50 details (by the Rayleigh criterion) on the surface of best focus which is a surface of rotation generated by the curve $y \sim x^{3/2}$.

(iii) On a plane surface the number of resolved image elements decreases to about one half the best focus surface value.

(iv) Increasing the aperture D/λ of a zone plate twofold and maintaining the same aperture ratio increases the number of image elements in the frame about 1.6-fold.

Relations have been derived that describe the operating range of a zone plate, and the Q-factor has been estimated for both axial and off-axis point sources.

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